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USE OF LAPLACE TRANSFORM & SUMUDU TRANSFORMS IN CRYPTOGRAPHY

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Abstract: In real life mathematics plays an important role in the process of cryptography.In this paper we introduced cryptographic method i.e. encryption and decryption method by using Laplace transform &Sumudu transform and their inverses.For operating online facilities password is required for confidentiality. Also in military services at every stage services, Indian police confidentiality are required.

Key-words: Laplace transform, Sumudu transform, Inverse Laplace transform, Inverse Sumudu transform, Cryptography, Encryption, Decryption.

Introduction: Cryptography is associated with the process of converting ordinary plain text into unintelligible text and vice-versa. The cryptography is based on mathematical concept and used in many applications like online banking, online purchasing, banking transactions cards, computer passwords, e-commerce transactions, e-Governing, SMS service, e-mails, ATM cards etc. In human life the security of financial information is an essential part. The purpose of using this method is for more security in communication as compared to other methods because cipher text obtained by this method could not be cracked by other persons easily. In the first part we apply Laplace transform to exponential function for Sumudu transform for the same purpose.Finally we conclude by comparing these two methods.

Preliminaries:

Definition:Laplace Transform: The Laplace transform of a function f(t) defined for all real numbers t ≥0, is the function F(s), which is a unilateral transform defined by

$L[f(t)] = F(s) = \int_0^\infty f(t)e^{-st}dt$ where s is a complex number frequency parameter

Formulae:$L[t^n] = \frac{n!}{s^{n+1}}$

Definition:Inverse Laplace Transform: If F F(s) is the Laplace transform of f(t) then the inverse Laplace transform of F(s) is given by f(t) and we write

$L^{-1}[F(s)] = f(t).$

Definition: Sumudu transform: Consider a set A defined as

$A = \left\{f(t) \mid \exists\, M, T_1, T_2 > 0, |f(t)| \leq Me^{\frac{|t|}{T_j}}, \text{if } t \in (-1)^j \times [0,\infty)\right\}$ For all real $t \geq 0$.

the Sumudu transform of a function f(t) ∈ A, is denoted by F(u) = S{f(t)] and is denoted as

$F(u) = S[f(t)] = \int_0^\infty \frac{1}{u} e^{-\frac{t}{u}} f(t)dt, u \in (-T_1, T_2)$

Definition :InverseSumudu transform: If F(u) is the Sumudu transform of f(t) then the inverse Sumudu transform of F(u) is f(t) and we write $f(t) = S^{-1}[f(t)]$

Formulae:

1) If $f(t) = t^n$ then $L[t^n] = \frac{n!}{s^{n+1}}$
2) If $f(t) = 1$ then $S[f(t)] = 1$
3) If $f(t) = t$ then $S[f(t)] = u$
4) If $f(t) = \frac{t^{(n-1)}}{(n-1)!}$, $n = 1,2,3,\cdots$ then $S[f(t)] = u^{(n-1)}$

Encryption: Suppose we want to send the massage "EXAMINATION"

We consider the Maclaurin's expansionof e^t given by

$$e^t = 1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \frac{t^6}{6!} + \frac{t^7}{7!} + \frac{t^8}{8!} + \frac{t^9}{9!} + \cdots = \sum_{k=0}^{\infty} \frac{t^k}{k!}$$

Using this expansion we write,

$$t^2 e^t = t^2 + \frac{t^3}{1!} + \frac{t^4}{2!} + \frac{t^5}{3!} + \frac{t^6}{4!} + \frac{t^7}{5!} + \frac{t^8}{6!} + \frac{t^9}{7!} + \frac{t^{10}}{8!} + \frac{t^{11}}{9!} + \cdots = \sum_{k=0}^{\infty} \frac{t^{k+2}}{k!} \qquad \text{------ (1)}$$

In this method we can convert the given plain text in to such a hidden text which could not Possible to crack without key by operating Laplace transforms. Suppose that we are given A B C D E F G H…………Z. as a plain text. In the first step we have to give the following allotment to letters in the given plain text.

A ↓ 0	B ↓ 1	C ↓ 2	D ↓ 3	E ↓ 4	F ↓ 5	G ↓ 6	H ↓ 7	I ↓ 8	J ↓ 9	K ↓ 10	L ↓ 11	M ↓ 12
N ↓ 13	O ↓ 14	P ↓ 15	Q ↓ 16	R ↓ 17	S ↓ 18	T ↓ 19	U ↓ 20	V ↓ 21	W ↓ 22	X ↓ 23	Y ↓ 24	Z ↓ 25

Since the plane text is "EXAMINATION" it is equivalent to 4,23,0,12,8,13,0,19,8,14,13.

Let us assume that

$C_0 = 4$, $C_1 = 23$, $C_2 = 0$, $C_3 = 12$, $C_4 = 8$, $C_5 = 13$, $C_6 = 0$, $C_7 = 19$, $C_8 = 8$, $C_9 = 14$, $C_{10} = 13$, and $C_k = 0$ for $k \geq 11$

Consider $f(t) = \sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!}$

$$= C_0 t^2 + C_1 \frac{t^3}{1!} + C_2 \frac{t^4}{2!} + C_3 \frac{t^5}{3!} + C_4 \frac{t^6}{4!} + C_5 \frac{t^7}{5!} + C_6 \frac{t^8}{6!} + C_7 \frac{t^9}{7!} + C_8 \frac{t^{10}}{8!} + C_9 \frac{t^{11}}{9!} + C_{10} \frac{t^{12}}{10!} + \cdots$$

$$= 4 \times t^2 + 23 \times \frac{t^3}{1!} + 0 \times \frac{t^4}{2!} + 12 \times \frac{t^5}{3!} + 8 \times \frac{t^6}{4!} + 13 \times \frac{t^7}{5!} + 0 \times \frac{t^8}{6!} + 19 \times \frac{t^9}{7!} + 8 \times \frac{t^{10}}{8!} + 14 \times \frac{t^{11}}{9!} + 13 \times \frac{t^{12}}{10!}$$

I)Taking Laplace transform on both sides we get,

$L[f(t)](u) = L\left[\sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!}\right](u)$

$= L\left[4 \times t^2 + 23 \times \frac{t^3}{1!} + 0 \times \frac{t^4}{2!} + 12 \times \frac{t^5}{3!} + 8 \times \frac{t^6}{4!} + 13 \times \frac{t^7}{5!} + 0 \times \frac{t^8}{6!} + 19 \times \frac{t^9}{7!} + 8 \times \frac{t^{10}}{8!} + 14 \times \frac{t^{11}}{9!} + 13 \times \frac{t^{12}}{10!}\right](u)$

$= 4 \times L(t^2)(u) + 23 \times L\left(\frac{t^3}{1!}\right)(u) + 0 \times L\left(\frac{t^4}{2!}\right)(u) + 12 \times L\left(\frac{t^5}{3!}\right)(u) + 8 \times L\left(\frac{t^6}{4!}\right)(u) + 13 \times L\left(\frac{t^7}{5!}\right)(u) + 0 \times L\left(\frac{t^8}{6!}\right)(u) + 19 \times L\left(\frac{t^9}{7!}\right)(u) + 8 \times L\left(\frac{t^{10}}{8!}\right)(u) + 14 \times L\left(\frac{t^{11}}{9!}\right)(u) + 13 \times L\left(\frac{t^{12}}{10!}\right)(u)$

$= 4\times \frac{2!}{s^3} + 23 \times \frac{3!}{1!\,s^4} + 0 \times \frac{4!}{2!\,s^5} + 12 \times \frac{5!}{3!\,s^6} + 8 \times \frac{6!}{4!\,s^7} + 13 \times \frac{7!}{5!\,s^8} + 0 \times \frac{8!}{6!\,s^9} + 19 \times \frac{9!}{7!\,s^{10}} + 8 \times \frac{10!}{8!\,s^{11}} + 14 \times \frac{11!}{9!\,s^{12}} + 13 \times \frac{12!}{10!\,s^{13}}$

$= 4\times \frac{2}{s^3} + 23 \times \frac{6}{s^4} + 0 \times \frac{12}{s^5} + 12 \times \frac{20}{s^6} + 8 \times \frac{30}{s^7} + 13 \times \frac{42}{s^8} + 0 \times \frac{56}{s^9} + 19 \times \frac{72}{s^{10}} + 8 \times \frac{90}{s^{11}} + 14 \times \frac{110}{s^{12}} + 13 \times \frac{132}{s^{13}}$

$= 8\times \frac{1}{s^3} + 138 \times \frac{1}{s^4} + 0 \times \frac{1}{s^5} + 240 \times \frac{1}{s^6} + 240 \times \frac{1}{s^7} + 546 \times \frac{1}{s^8} + 0 \times \frac{1}{s^9} + 1368 \times \frac{1}{s^{10}} + 720 \times \frac{1}{s^{11}} + 1540 \times \frac{1}{s^{12}} + 1716 \times \frac{1}{s^{13}}$ ------------ (2)

Let us calculate C_i such that $r_i \equiv C_i \bmod 26$

i.e. $8\equiv -18 \bmod 26$, $138\equiv 8 \bmod 26$, $0\equiv 0 \bmod 26$, $240\equiv 6 \bmod 26$,

$240\equiv 6 \bmod 26$, $546 \equiv 0 \bmod 26$, $0\equiv 0 \bmod 26$, $1368\equiv 16 \bmod 26$,

$720 \equiv 18 \bmod 26$, $1540\equiv 6 \bmod 26$, $1716\equiv 0 \bmod 26$

Let $C_0 = -18$, $C_1 = 8$, $C_2 = 0$, $C_3 = 6$, $C_4 = 6$, $C_5 = 0$, $C_6 = 0$, $C_7 = 16$, $C_8 = 18$, $C_9 = 6$, $C_{10} = 0$, and the quotients 1, 5, 0, 9, 9, 21, 0, 52, 27, 59, 66 form the key.

Using the assignment given in equation (1) the cipher text for given plain text "EXAMINATION" will be -18, 8, 0, 6, 6, 0, 0, 16, 18, 6, 0 i.e. "SIAGGAAQSGA". The sender publicity sends the message "SIAGGAAQSGA" and $\sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!}$ privately sends the key and the Sumudu transform.

Decryption: the receiver receives the message "SIAGGAAQSGA".

The equivalent values are

S, I, A, G, G, A, A, Q, S, G, A.

-18, 8, 0, 6, 6, 0, 0, 16, 18, 6, 0.

and the private key values are 1, 5, 0, 9, 9, 21, 0, 52, 27, 59, 66.

Since $C_k = 26 \times \text{key} + \text{remainder}$, i.e.

$8 = 26 \times 1 - 18$, $138 = 26 \times 5 + 8$, $0 = 26 \times 0 + 0$,

$240 = 26 \times 9 + 6$, $240 = 26 \times 9 + 6$, $546 = 26 \times 21 + 0$,

$0 = 26 \times 0 + 0$, $1368 = 26 \times 52 + 16$, $720 = 26 \times 27 + 18$,

$1540 = 26 \times 59 + 6, 1716 = 26 \times 66 + 0$.

Thus we get 8, 138, 0, 240, 240,546,0,1368, 720,1540,1716 which implies

$$\sum_{k=0}^{\infty} C_k \frac{(k+2)!u^{k+2}}{k!} = 8\times\frac{1}{s^3} + 138\times\frac{1}{s^4} + 0\times\frac{1}{s^5} + 240\times\frac{1}{s^6} + 240\times\frac{1}{s^7} + 546\times\frac{1}{s^8} + 0\times\frac{1}{s^9} + 1368\times\frac{1}{s^{10}} + 720\times\frac{1}{s^{11}} + 1540\times\frac{1}{s^{12}} + 1716\times\frac{1}{s^{13}}$$

$$= 4\times\frac{2}{s^3} + 23\times\frac{6}{s^4} + 0\times\frac{12}{s^5} + 12\times\frac{20}{s^6} + 8\times\frac{30}{s^7} + 13\times\frac{42}{s^8} + 0\times\frac{56}{s^9} + 19\times\frac{72}{s^{10}} + 8\times\frac{90}{s^{11}} + 14\times\frac{110}{s^{12}} + 13\times\frac{132}{s^{13}}$$

$$= 4\times\frac{2!}{s^3} + 23\times\frac{3!}{1!\,s^4} + 0\times\frac{4!}{2!\,s^5} + 12\times\frac{5!}{3!\,s^6} + 8\times\frac{6!}{4!\,s^7} + 13\times\frac{7!}{5!\,s^8} + 0\times\frac{8!}{6!\,s^9} + 19\times\frac{9!}{7!\,s^{10}} + 8\times\frac{10!}{8!\,s^{11}} + 14\times\frac{11!}{9!\,s^{12}} + 13\times\frac{12!}{10!\,s^{13}}$$

Taking inverse Laplace transform on both sides we get

$$L^{-1}\left[\sum_{k=0}^{\infty} C_k \frac{(k+2)!\,u^{k+2}}{k!}\right] = L^{-1}\left[4\times\frac{2!}{s^3} + 23\times\frac{3!}{1!\,s^4} + 0\times\frac{4!}{2!\,s^5} + 12\times\frac{5!}{3!\,s^6} + 8\times\frac{6!}{4!\,s^7} + 13\times\frac{7!}{5!\,s^8} + 0\times\frac{8!}{6!\,s^9} + 19\times\frac{9!}{7!\,s^{10}} + 8\times\frac{10!}{8!\,s^{11}} + 14\times\frac{11!}{9!\,s^{12}} 13\times\frac{12!}{10!\,s^{13}}\right]$$

$$\sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!} = 4\times t^2 + 23\times\frac{t^3}{1!} + 0\times\frac{t^4}{2!} + 12\times\frac{t^5}{3!} + 8\times\frac{t^6}{4!} + 13\times\frac{t^7}{5!} + 0\times\frac{t^8}{6!} + 19\times\frac{t^9}{7!} + 8\times\frac{t^{10}}{8!} + 14\times\frac{t^{11}}{9!} + 13\times\frac{t^{12}}{10!} \quad \text{-----------} \ (3)$$

In the above expansion the coefficients C_k are 4, 23, 0, 12, 8, 13, 0, 19, 8, 14, 13, which gives the original plain text message as ,

4, 23, 0, 12, 8, 13, 0, 19, 8, 14, 13.

E, X, A, M, I, N, A, T, I, O, N.

II) Taking Sumudu transform on both sides we get,

$$S[f(t)](u) = S\left[\sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!}\right](u)$$

$$= S\left[4\times t^2 + 23\times\frac{t^3}{1!} + 0\times\frac{t^4}{2!} + 12\times\frac{t^5}{3!} + 8\times\frac{t^6}{4!} + 13\times\frac{t^7}{5!} + 0\times\frac{t^8}{6!} + 19\times\frac{t^9}{7!} + 8\times\frac{t^{10}}{8!} + 14\times\frac{t^{11}}{9!} + 13\times\frac{t^{12}}{10!}\right](u)$$

$= 4\times S(t^2)(u) + 23\times S\left(\frac{t^3}{1!}\right)(u) + 0\times S\left(\frac{t^4}{2!}\right)(u) + 12\times S\left(\frac{t^5}{3!}\right)(u) + 8\times S\left(\frac{t^6}{4!}\right)(u) + 13\times S\left(\frac{t^7}{5!}\right)(u) + 0\times S\left(\frac{t^8}{6!}\right)(u) + 19\times S\left(\frac{t^9}{7!}\right)(u) + 8\times S\left(\frac{t^{10}}{8!}\right)(u) + 14\times S\left(\frac{t^{11}}{9!}\right)(u) + 13\times S\left(\frac{t^{12}}{10!}\right)(u)$

$= 4\times 2!\,u^2 + 23\times 3!\frac{u^3}{1!} + 0\times 4!\frac{u^4}{2!} + 12\times 5!\frac{u^5}{3!} + 8\times 6!\frac{u^6}{4!} + 13\times 7!\frac{u^7}{5!} + 0\times 8!\frac{u^8}{6!} + 19\times 9!\frac{u^9}{7!} + 8\times 10!\frac{u^{10}}{8!} + 14\times 11!\frac{u^{11}}{9!} + 13\times 12!\frac{u^{12}}{10!}$

$= 8\times u^2 + 138\times u^3 + 0\times u^4 + 240\times u^5 + 240\times u^6 + 546\times u^7 + 0\times u^8 + 1368\times u^9 + 720\times u^{10} + 1540\times u^{11} + 1716\times u^{12}$--------- (4)

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i.e. $8\equiv -18 \bmod 26$, $138\equiv 8 \bmod 26$, $0\equiv 0 \bmod 26$, $240\equiv 6 \bmod 26$,

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$720\equiv 18 \bmod 26$, $1540\equiv 6 \bmod 26$, $1716\equiv 0 \bmod 26$

Let $C_0 = -18$, $C_1 = 8$, $C_2 = 0$, $C_3 = 6$, $C_4 = 6$, $C_5 = 0$, $C_6 = 0$, $C_7 = 16$, $C_8 = 18$, $C_9 = 6$, $C_{10} = 0$, and the quotients 1, 5, 0, 9, 9, 21, 0, 52, 27, 59, 66 form the key.

Using the assignment given in equation (1) the cipher text for given plain text “EXAMINATION” will be -18, 8, 0, 6, 6, 0, 0, 16, 18, 6, 0 i.e. “SIAGGAAQSGA”. The sender publicity sends the message “SIAGGAAQSGA” and $\sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!}$ privately sends the key and the Sumudu transform.

Decryption: the receiver receives the message “SIAGGAAQSGA”.

The equivalent values are

S, I, A, G, G, A, A, Q, S, G, A.

-18, 8, 0, 6, 6, 0, 0, 16, 18, 6, 0.

and the private key values are 1, 5, 0, 9, 9, 21, 0, 52, 27, 59, 66.

Since $C_k = 26\times$ key + remainder, i.e.

$8 = 26\times 1 - 18$, $138 = 26\times 5 + 8$, $0 = 26\times 0 + 0$,

$240 = 26\times 9 + 6$, $240 = 26\times 9 + 6$, $546 = 26\times 21 + 0$,

$0 = 26\times 0 + 0$, $1368 = 26\times 52 + 16$, $720 = 26\times 27 + 18$,

$1540 = 26\times 59 + 6$, $1716 = 26\times 66 + 0$.

Thus we get 8, 138, 0, 240, 240,546,0,1368, 720,1540,1716 which implies

$\sum_{k=0}^{\infty} C_k \frac{(k+2)!u^{k+2}}{k!} = 8 \times u^2 + 138 \times u^3 + 0 \times u^4 + 240 \times u^5 + 240 \times u^6 + 546 \times u^7 + 0 \times u^8 + 1368 \times u^9 + 720 \times u^{10} + 1540 \times u^{11} + 1716 \times u^{12}$

$= 4 \times 2!\,u^2 + 23 \times 3!\frac{u^3}{1!} + 0 \times 4!\frac{u^4}{2!} + 12 \times 5!\frac{u^5}{3!} + 8 \times 6!\frac{u^6}{4!} + 13 \times 7!\frac{u^7}{5!} + 0 \times 8!\frac{u^8}{6!} + 19 \times 9!\frac{u^9}{7!} + 8 \times 10!\frac{u^{10}}{8!} + 14 \times 11!\frac{u^{11}}{9!} + 13 \times 12!\frac{u^{12}}{10!}$

Taking inverse Sumudu transform on both sides we get

$S^{-1}\left[\sum_{k=0}^{\infty} C_k \frac{(k+2)!\,u^{k+2}}{k!}\right] = S^{-1}\left[4 \times 2!\,u^2 + 23 \times 3!\frac{u^3}{1!} + 0 \times 4!\frac{u^4}{2!} + 12 \times 5!\frac{u^5}{3!} + 8 \times 6!\frac{u^6}{4!} + 13 \times 7!\frac{u^7}{5!} + 0 \times 8!\frac{u^8}{6!} + 19 \times 9!\frac{u^9}{7!} + 8 \times 10!\frac{u^{10}}{8!} + 14 \times 11!\frac{u^{11}}{9!} + 13 \times 12!\frac{u^{12}}{10!}\right]$

$\sum_{k=0}^{\infty} C_k \frac{t^{k+2}}{k!} = 4 \times t^2 + 23 \times \frac{t^3}{1!} + 0 \times \frac{t^4}{2!} + 12 \times \frac{t^5}{3!} + 8 \times \frac{t^6}{4!} + 13 \times \frac{t^7}{5!} + 0 \times \frac{t^8}{6!} + 19 \times \frac{t^9}{7!} + 8 \times \frac{t^{10}}{8!} + 14 \times \frac{t^{11}}{9!} + 13 \times \frac{t^{12}}{10!}$ --------- (5)

In the above expansion the coefficients C_k are 4, 23, 0, 12, 8, 13, 0, 19, 8, 14, 13, which gives the original plain text message as ,

4, 23, 0, 12, 8, 13, 0, 19, 8, 14, 13.

E, X, A, M, I, N, A, T, I, O, N.

Conclusion: In this work a new cryptographic application is introducedby applying Laplace and Sumudu transform to the same function we will obtain the same cipher text for the given plain text. The private key is the quotient of the C_n when divided by 26,the number of multiples of mod 26.It is very difficult to find the private key by any other attack. After producing key we use this key for encryption and decryption that algorithm based on Laplace and Sumudu transformation and modular arithmetic. The same result can be found by considering Laplace and Sumudu transform of hyperbolic sine and cosine functions, trigonometric sine and cosine as well as in polynomial function.

References:

1) Hemant K. Undegaonkar, R. N. Ingale : Role of Some Integral Transforms in Cryptography, International Journal of Engineering and Advanced Technology , Volume – 9 Issue – 3 Feb, Issue – 3 Feb,2020.
2) Bodkhe D. S. and Panchal S. K. : Use of Sumudu transform in cryptography, Bulletin of the Marathawada Mathematical Society Vol.16, No. 1, June 2015, page 1- 6.
3) Hiwarekar A. P. : A new method of Cryptography using Laplace transform of hyperbolic function, Inter. J. Math.Arch.,4(2), 208- 213,(2013).
4) Barr T, H. : Invitation to Cryptography, Prentice Hall, (2002).
5) Tarig M. Elzaki and Salih M. Ezaki : On the connection between Laplace and Elzaki transforms, Advances in theoretical and Applied Mathematics Volume 6, Number 1(20110,pp. 1-10.
6) Stallings W.: Cryptography and network Security, Prentice Hall, (4th edition)(2005).

7) Watungla G. K. : Sumudu Transform- an integral transform to solve differential equation and control engineering Education in Science and Technology, 24(1)problems, International Journal of Mathematical Education in science and technology, 24(1), 35 – 43, (1993).
8) Hassan Eltayeb and Adem Kilicman : On some Applications of a New Integral Transform, Int. Journal of Math. Analysis, Vol. 4, 2010, no. 3, 123 – 132.